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Скрытые фигуры и потаенная геометрия в Прелюдии А. Н. Скрябина op. 74 № 1

Аннотация

В данной статье Прелюдия А. Н. Скрябина op. 74 № 1 рассматривается в новом свете. В качестве отправной точки используется исследование связи между музыкой и математикой, демонстрируется, как посредством геометрических фигур гармонические функции могут быть представлены графически. Более подробное рассмотрение Прелюдии включает в себя анализ гармонии: в пьесе используется нескольких типов шестинотных аккордов. Эти аккорды представлены в виде гексахордов и изучены как геометрические фигуры. Таким образом, устанавливается связь между гармонией и фигурами. Прослеживаются исторические аналогии и возможные перспективы новых исследований в аспекте «гармонии мира».

Ключевые слова

Александр Скрябин, прелюдия, связь математики и музыки, гармония, представление гармонии через геометрические фигуры, гексахорды

Latent figures and recondite geometries in Scriabin's Prelude op. 74 no. 1

Abstract

This paper considers Scriabin's Prelude op. 74 no. 1 in a new light. Starting from studies on the relation between music and mathematics, it shows how harmonic functions in tonal language can be represented graphically using geometric figures. It then goes deeper into the prelude. The harmonies of the piece are taken into consideration: they are made up of several types of six-note chords. These chords are represented as hexachords and studied as geometrical figures. Thus, a link between harmonies and figures is created. Historical analogies and possible prospects for new research in the aspect of 'harmony of the world' are traced.

Keywords

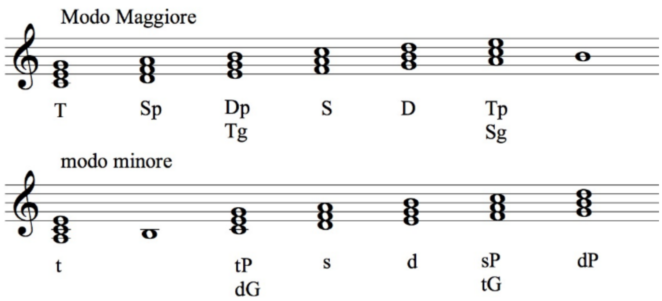
Alexander Scriabin, prelude, the relation between mathematics and music, harmony, representation of harmonies through geometrical figures, hexachords

Within the field of studies of the relationships between mathematics and music, the geometric representation of chords has been much investigated. Geometric figures have been linked to chords to represent them visually. Thus, a strict relationship between what can be heard and what can be seen has been established.

Among many contributions, Dmitri Tymoczko's research is fundamental in this regard. On several occasions, he has not only shown a way to represent an aggregate of sounds in three dimensions,[1] but also how to represent connections among chords that have a common voice or that proceed chromatically.[2] This study builds upon what has been done so far and seeks to show how certain geometric figures, derived from the analysis of Prelude op. 74 no. 1, can 'sound'. In this sense, such a study highlights another dimension of music. That is to say, a sort of visual appreciation of the score, which is not referred to the score properly written, but rather to a representation of what one can find on in the score.

It is possible to represent harmonic functions as geometric figures, that is to say: to see which figures are obtained from the codification of harmonic functions in another codex. In fact, in this case, chords will be linked to figures in the spaces. Thus, what is heard in a mental space will be represented in a graphic space, which will not be the score. To do this, let us consider the classification used by Diether De La Motte in his book "Theory of Harmony". Specifically, De La Motte distinguishes between main functions, parallel chords, and counterchords. In the major mood, the three main functions correspond to the three major chords built on the I, IV, and V grades. In the minor mood, they correspond to the three minor chords, always on the same grade. The parallel chords are those whose fundamental is a minor third below the corresponding main harmonic function. On the other hand, the counter-chords are those whose fundamental is a major third above the corresponding harmonic function. The scheme, taken from De La Motte's introduction, clearly shows the chord classification in the major and minor moods (Example 1).

Example 1. Chord classification in the major and minor moods according to



D. De La Motte [3, *24*].

It is immediately evident that some chords are both parallel and counter-chords. Specifically, with regard to the major mode, in Example 1, 'T' indicates the tonic, 'S' – the subdominant, and 'D' – the dominant. 'Sp' indicates the minor parallel chord of the subdominant, 'Dp' – the minor parallel chord of the dominant, and 'Tp' – the minor parallel chord of the major tonic. 'Tg' indicates the minor counter-chord of dominant, and 'Sg' indicates the minor counter-chord of subdominant. Regarding the minor mode, the main functions are indicated by lower case lowercase letters:[3] 'tP' indicates a major parallel chord of minor tonic;

[1] In a general sense, an aggregate of sound can be a chord made with thirds but also with seconds and other intervals.

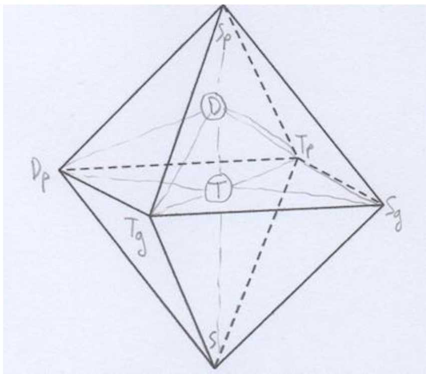
[2] Cf. studies by Tymoczko ([12, *72–74*], [13], [14, *1–52*]).

[3] In Example 1, it is clear that 'd', indicating the dominant, makes no sense because the dominant is exclusively a major chord. Thus, it should be indicated with a 'D'. In the example, the natural minor scale is taken into consideration, therefore, 'd' refers to the minor chord built on the fifth degree of the natural minor scale.

'sP' – a parallel chord greater than subdominant (minor); 'dG' – a major counter-chord of dominant, and 'G' – a major counter-chord of minor tonic.

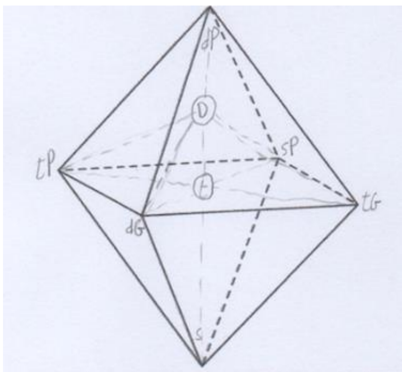
Defining the tonality as a set of relations between irreversible hierarchical structures, in order to represent this conception in a three-dimensional way, we can use an octahedron whose center is the fundamental harmonic relation for the tonality in general, which is the triton. (The Tonic, being the most important function of the tonal system, is at the center of the figure. The D, being a fifth above, is above the T in the figure, and going on, the S, being a fifth below, is below the T. The others are peripheral functions, so in the octahedron, they are close to the borders. But all of them are related (in connection with the T). Maybe to be even clearer S could be a bit closer to T in the figure.)

The other functions can be found around the vertices of the octahedron.[4] (For the case of the major mode see: Example 2.)



Example 2. Octahedral scheme of chord relations in the major mood.

In terms of space, the tonic (C) and the dominant (G) are found inside the polyhedron. At the bottom corner is the subdominant (F, indicated also as counter-dominant or inferior dominant), and at the upper corner is the parallel chord of the subdominant (D). On the sides at the same time the parallel chord and counter-chord and the E that is in the same situation.[5] (For the case of the minor mode see: Example 3).



Example 3. Octahedral scheme of chord relations in the minor mood.

[4] It is a polyhedron known since ancient times. It is one of the five Platonic solids, that is, convex polyhedra whose faces are regular and equal convex polygons.

[5] For a focus upon the stability for chords written in geometric figures, cf. [8, *201*].

The succession of harmonic functions of a composition will, therefore, be represented by a set of figures that will develop and follow one another within the octahedron depending upon the harmonic functions of the piece. For example, the first eight bars of Mozart's Piano Sonata K 282[6] are shown in Example 4. Example 5 shows a scheme of the harmonic functions of a fragment of the piece, the first eight bars.[7]

Example 4. W. A. Mozart. Piano Sonata K 282. First eight bars.

T D^6 Tp DD I D I D^2 T^6 D^7 I T I D I T^6 t I DD^7 (DD)-D I DD^7 I

Example 5. Harmonic functions of the first eight bars of Mozart's Piano Sonata K 282.

In Example 5, the penultimate bar contains the symbol ('DD') that indicates the dominant of the dominant.[8] At this point it is possible to show the chain of the harmonic succession of the piece inside the octahedron and see which geometrical figures are created. Of course, the result will be different figures on each bar. The type of figures that will be obtained will depend upon the number of harmonic functions contained in the bar and their specificity. Drawing this type of chart for the first eight bars of the sonata, it is interesting to see how the figures change according to the different harmonic sequences and then compare them. This exhaustive study is beyond the scope of this paper. However, by way of example, the first line will be considered here. Starting from the assumption that the position of the chord is not considered in this type of analysis but the harmonic function is, the points affected will be T, D, Tp, and DD. The first three will be shown within the figure. As DD does not belong to the number of chords of the mode, it will be represented externally to the octahedron (Example 6). By joining the points, you get a figure that represents this sequence.

Thus, it could almost be said that the figures in the octahedron 'sound' like the sound that we hear by performing the lines of the piece. In this sense, it is possible to appreciate a correspondence between geometric figures and the sound of the score.

In an atonal or wide-tonality environment, the situation is different, as in the case of the composition considered here. Being atonal, it will be necessary to study a system to visually

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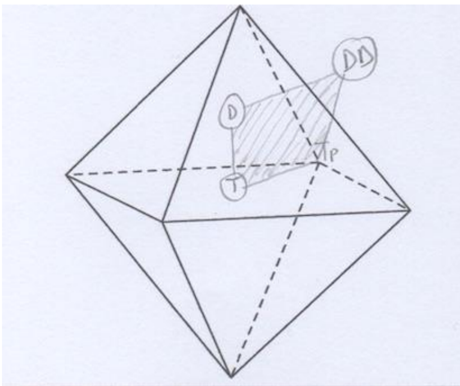
[6] The example shows the beginning of the piece. Thus, the first eight bars in the example are the first eight bars of the piano sonata.

[7] Bold vertical lines indicate the bars. Thus, we understand the harmonic functions for each.

[8] De La Motte, Manuale di Armonia, [3, 160].

represent the chromatic set, which will be different from that presented above that could represent a tonal environment.

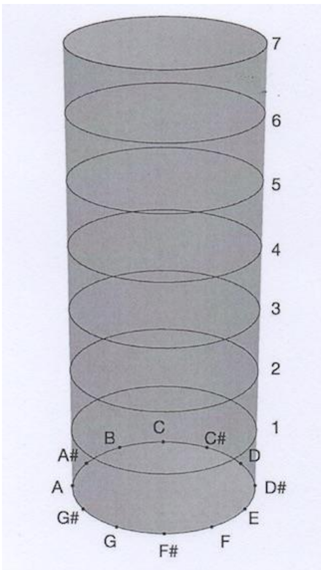


Example 6. Position of the dominant of the dominant (DD) and the sequence with T, D, Tp, and DD in the octahedral scheme of chord relations.

Thus, the graphic situation will change a lot if we consider other types of language, such as atonal language. This is the case with Scriabin's Prelude op. 74 no. 1, which was written in 1914, a year before the composer's death. There are many studies of the harmony of this piece and on voice leading in Scriabin in general.[9]

For the purposes of this paper, the scheme representing the harmonic functions exposed in Example 1 and Example 2 will be absolutely insufficient. In fact, given that the piece is not tonal in the strict sense, it will no longer be possible to talk about classical harmonic functions, but it will be necessary to consider the set of 12 sounds of the chromatic scale out of classical hierarchy. Furthermore, the harmonic functions remain the same regardless of the diastematic space, but in the non-tonal environment a note placed at one octave or another has a completely different value.

For all these reasons, it is clear that another type of figure is needed. If one rotates the octahedron based on the horizontal axis and makes it rise, so as to describe a diastematic space, it is possible to obtain a cylinder. This figure seems best for depicting figures in the atonal environment (Example 7).

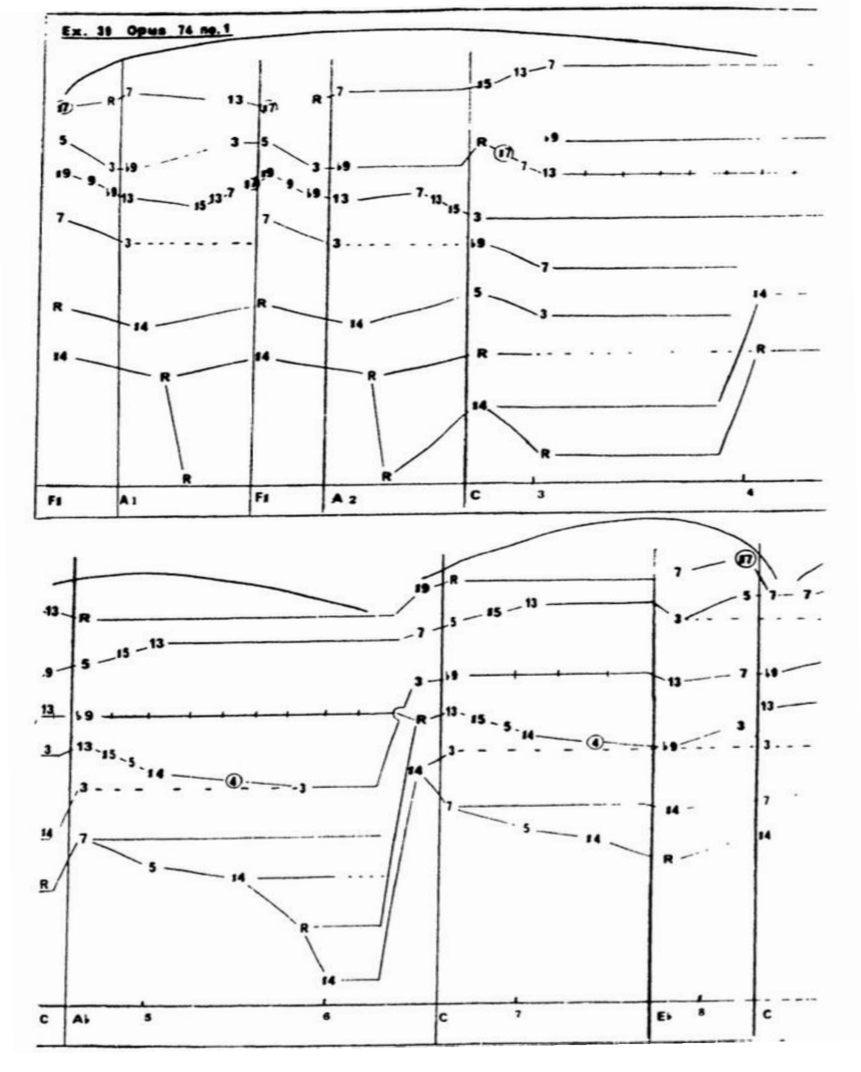


Example 7. Cylindrical model of the tonal space.

[9] For these aspects, the reader will find a sizeable bibliography.

The circle constitutes the base. Then 12 points have been drawn. They correspond to the notes of the chromatic scale. As in a clock, for example, the first is *C*, and the others proceed going up by a semitone on the right. The circles above the base represent the octaves of the piano. If we consider the central *C* of the piano as *C4*, the circle 4 represents the central octave.[10] So, depending on the diastematic region in which the note of the chord is set, it will be drawn on one circle or another. Example 7 shows the cylinder as a reference system for drawing the figures, as described above.

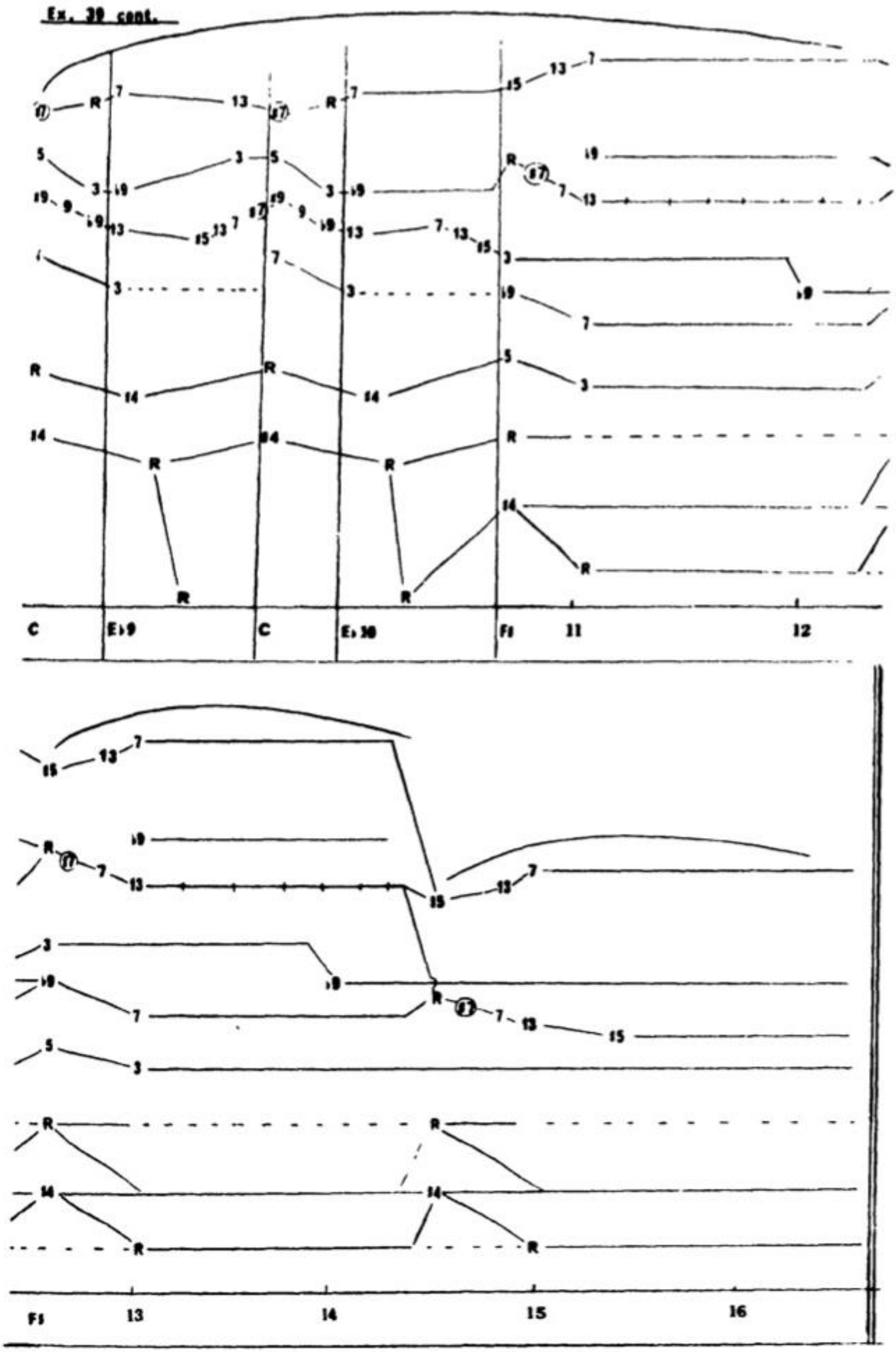
To analyze the chords of the Prelude, reference will be made to those proposed by Roderick Shergold and reported in Example 8.[11] The analysis shows a latent, almost implied tonic, which is *Fis*. The scheme of Example 8 clearly shows this 'hidden' tonic that is not perceived but emerges from analysis of the score. In this Example, the roots of chords are indicated in the boxes below the lines. They are on the right of a vertical line that indicates a change in harmony. The numbers above indicate the number of bars. The horizontal broken lines indicate the movements of the voices and the numbers indicates the notes of the chord.



Example 8. Roderick Shergold's analysis of Scriabin's Prelude op. 74 no. 1 [11, 88–97].

[10] In Europe, central *C* is indicated as C_3.

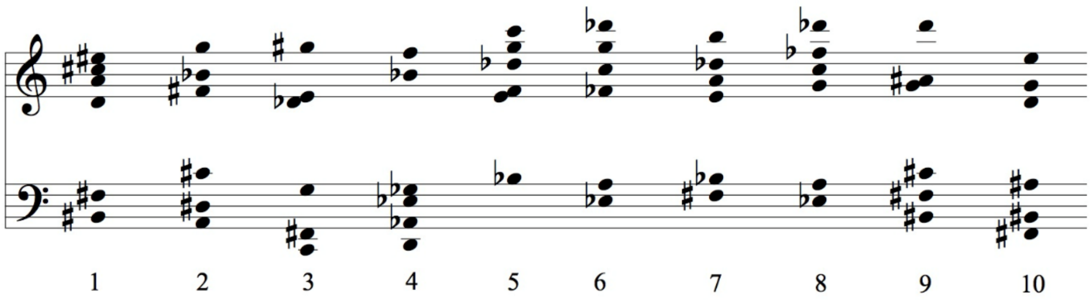
[11] I am very grateful to the professor for permission to reproduce the scheme of Example 8.



Example 8 (second part).

As we notice, the dense motion of the parts (voice leading) and the chordal hypertrophy make perception of a clear and stable tonic impossible. However, the example shows that there are chords that recur, or rather, they occur in the same position and then depart to generate others. It is a matter of starting from one point and then going in different directions. For this reason, the figures that we have will be the same, that is to say, they will be repeated, and the results, that is, the figures generated inside them, will be different. Depending on the movement of the parts as the piece progresses, the chords, and consequently the figures, are transformed into each other without interruption. For this reason, it would be very useful (and perhaps not even too ideal) to create an animation that shows how figures change, and shows how they change into each other as the piece proceeds.

To schematize, then, the chords in the position were considered the first time they occurred at the change of harmony. Example 9 shows these chords. The designation no. 1 is *C*, the first chord of the piece, while no. 2 is *A*.[12] These two chords are repeated, in the same position, in the first two bars of the Prelude. At the end of bar 2, the harmony of *C* (no. 3) appears, followed by *As* at bar 4 (no. 4). The harmony of *C* returns again to bar 6 (no. 5) followed by *Es* in bar 7 (no. 6). The designation no. 7 represents a variation of *C* and no. 8 of *Es*. The designations no. 9 and no. 10 are two positions of the harmony of *Fis* which is kept for a long time at the end of the piece (from the bar 10 to the end).

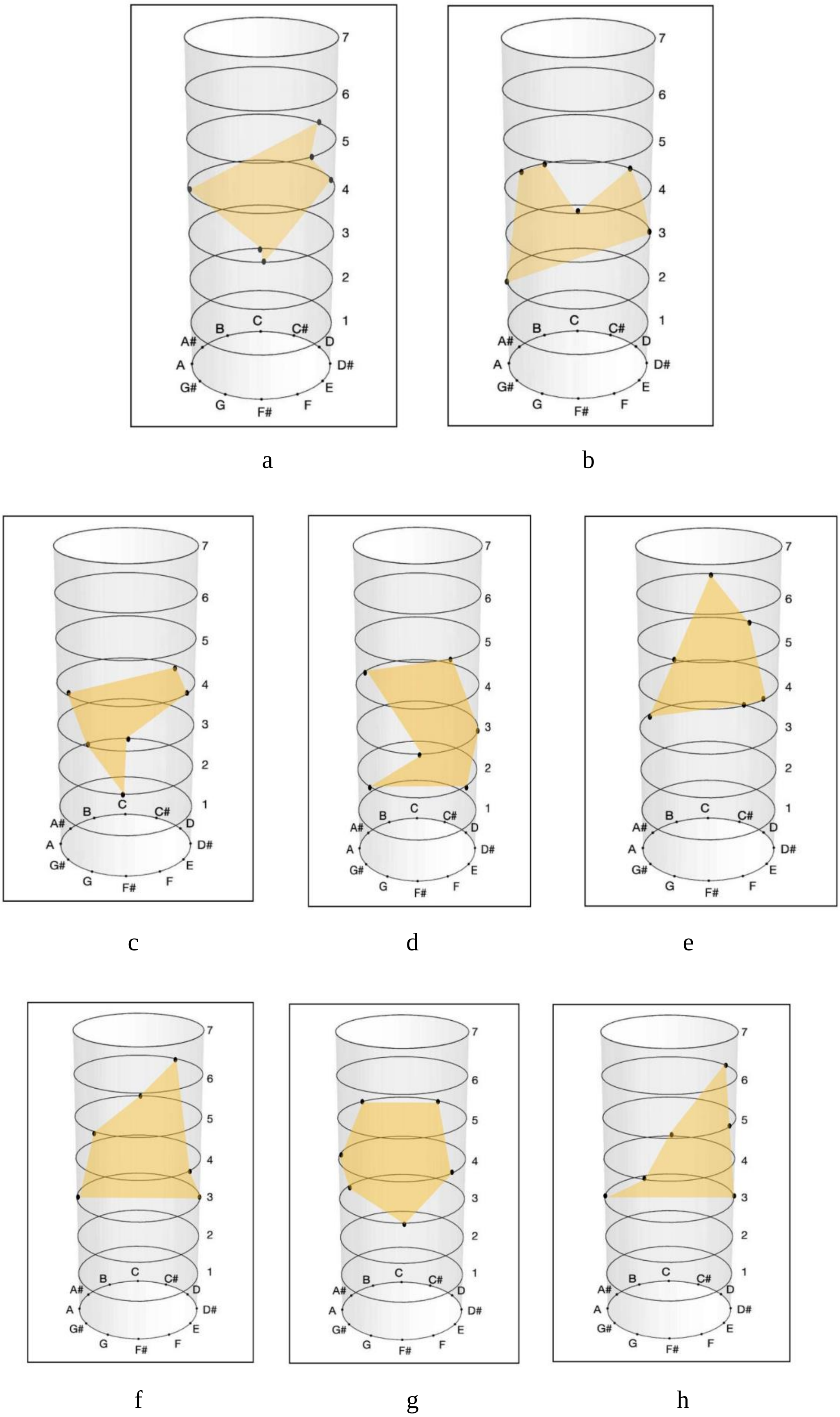


Example 9. Main chords in Scriabin's Prelude op. 74 no. 1.

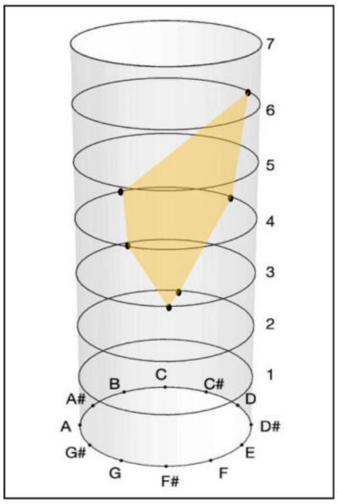
It is obvious that the diastematic ambit in which the harmonies develop is very broad and large. It is also interesting to note that, by repeating chords in the same position, we recognize the principle of cyclicality, even in a non-classical tonal context and even if it is not possible to perceive it clearly; it is a sort of recurring issue for different musical cultures and different languages. The chords described do not take into account the notes doubled in the same harmony. Harmonies composed of six tones come out. Therefore, as far as the generated figures are concerned, they are irregular hexagons. The irregularity of the figures depends on the fact that there are intervals of various sizes within the chords themselves. The degree of irregularity of what is seen in the cylinder is directly proportional to the greater or lesser degree of stability of the chord. Naturally, when drawing the figures in the examples below, enharmonic sounds are taken into account. The designations (Example 10, a and b) show the first chords of the piece: Fis, e, A. What is obtained, as with the following figures, are irregular surfaces, that is, surfaces with different angles, and while the sides are of different lengths, which take shape in the volume of the cylinder.

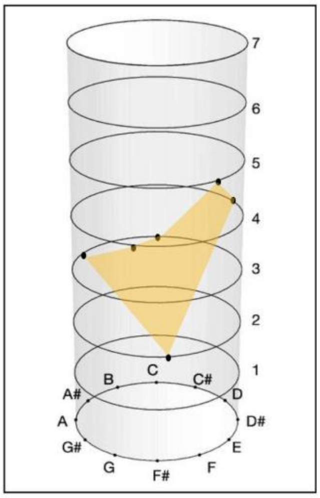
In this context, it will not be so useful to understand which is the fundamental of a chord, but to understand which are the notes of the chord itself and see the figure that is generated. In fact, what we study is the variation of the geometric figure that is formed. From the previous example, and from the next, we clearly see the irregular hexagons. The rest of Example 10 (from c to j) shows the other chords of the piece. Obviously, the figures generated by chords that are repeated and have the same position are not repeated. More specifically, Example 10 c (see above) represents the chords of no. 3, 10 d – no. 4, 10 e – no. 5, 10 f – no. 6, 10 g – no. 7, 10 h – no. 8, 10 i – no. 9, and finally 10 j – no. 10.

[12] The number indicates the position of chords in Example 9.



Example 10. Configuration of main chords of Scriabin's Prelude op. 74 no. 1 in cylindrical tonal space.





i j

Example 10 (second part).

In mathematics, the method used to calculate the minimum surface is called Plateau's Problem. To apply this to the figures described above can usefully be tackled in the future.

The role of polyhedrons in art and nature has long fascinated the most inquisitive minds in human civilization. Pythagoras and Plato, Kepler and Durer—all of them, each in their own research field, proved the leading role of geometry and mathematics in different hypostases of human existence: living nature and culture, cosmogony, and painting.

All these far-reaching analogies are in the tradition of Italian musicology. The composer S. Sciarrino, in his book "Figures of Music. From Beethoven to the Present Day" also draws parallels between similar configurations in nature and in music, writing about the existence of models common to different kinds of thinking activity models (embodied in creativity).

In this article, we wanted to draw attention to the following: The hexagonal shape is one of the most common shapes in nature. It is the shape of mollusks and frozen volcanic eruptions, the surface of a dragonfly's eye, and the structure of bee honeycombs. Do all polygons have the property of having "structures of nature that have an inner sound"? (We must also consider the fact that humans are unable to pick up this sound; the range of human audibility is extremely small.) So, what effect do these sounds have on us, even if we can't hear them? Are all humans immersed in an eternal sound that we cannot hear but from which we draw strength? What if these "inaudible" but imagined figures, pulsating in their structure, are patterns? Do these patterns not create an order that humanity has yet to discover?

We have only posed the problem without claiming to have solved it.

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